Introduction to Numerical Weather Prediction

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Abstract—Numerical weather prediction is an essential method for meteorologists to forecast the weather and lead to more precision in forecasting. Since the weather is a complex and chaotic system, numerical weather prediction also displays a high complexity. Therefore, this article gives an introduction for people without any or just little knowledge on the topic through illuminating the historical and conceptual aspects of numerical weather prediction. Additionally, simple verification methods are explained in order to examine the accuracy of weather forecasts. If a reader is more interested in one of the aspects he can use the gathered literature references to gain deeper knowledge.

I. INTRODUCTION

Although most people do not know how numerical weather prediction works, they are highly interested in the results, presented by TV weathermen or accessible via the Internet. Before people leave their homes they check the weather forecast to decide whether they need to take an umbrella with them or not. Aeroplanes and ships need weather forecasts to navigate safely. The importance of weather forecasts can also be seen in a historical context. Actually, weather forecasts were one reason for the success of the invasion of Allied troops on D-Day [1].

Computer scientists, physicians, mathematicians and meteorologists combine their knowledge to achieve better forecasts which is one indicator of the complexity of this research area. This paper simplifies the access to this broad field, it shows the origin of numerical weather prediction, the concept behind it and examines the accuracy of modern weather forecasts.

II. FUNDAMENTALS OF METEOROLOGY

For a better understanding of weather forecasting one should know about the fundamental processes which have an impact on the weather. However, this is not the purpose of this paper. The British meteorological service Met Office gives a good introduction for beginners [2]. Advanced learners should have a look on [3].

III. HISTORY

In order to enable numerical weather prediction, a lot of scientific developments were needed. Meteorology had to become a science, so that humans were able to understand the physical processes which make the weather. Furthermore numerical methods and computers had to be invented to enable numerical simulations.

Already Aristotle wrote a book about meteorology [4]. In 1609 Johannes Kepler calculated the orbit of the planets [5]. This means that he could predict the future position of two planets by knowing their current position and momentum. 1686 Edmond Halley, famous by Halley’s Comet, could explain the development of the trade winds by solar heating [6]. Figure 1 shows an experiment which demonstrates this effect. If you conduct this experiment you will see the flames of the candles turning slightly to the centre. This is caused by wind compensating the air pressure gradient between the high and the low above the ground.

George Hadley added earth’s rotation as a basic factor of wind in 1735 [7]. As one of the first scientists, Joseph Louis Lagrange described the idea of numerical simulations in 1759, but there was no practical use for his idea at this time [8]. Certainly one reason for this was that no computers existed. In 1835 William Ferrel postulated the importance of the Coriolis force for the weather processes [9].

Through observing the influence of an unknown planet on the orbit of Uranus Urban Le Verrier calculated that Neptune had to exist and he could make assumptions about its orbit and position. When Neptune was discovered in 1848 [10], this proved that physics and mathematics could be used to predict the development of physical processes [8]. Le Verrier also made another, more direct contribution to meteorology. He developed a professional concept for weather forecasts in 1854 [11].

The breakthrough came in 1904 when Vilhelm Bjerknes published his famous paper “Das Problem der Wettervorsage: betrachtet vom Standpunkte der Mechanik und der Physik” [12]. He combined all sciences which were needed to...
conduct numerical weather predictions: meteorology, physics and numerical mathematics.

It took another 46 years until his ideas could be charged into practice. In 1950 scientists could implement the first weather model on John von Neumann’s computer ENIAC [13].

IV. CONCEPT

The target of numerical weather simulations is to calculate the state of atmosphere depending on time. That means the simulation has the purpose to calculate the velocity, density, pressure, temperature and humidity of every single point in the air [12]. As it is not possible to regard every single point because of observational and computational limitations a two-dimensional or a three-dimensional grid is used for approximation [14].

A. Approach

You can divide the development of numerical simulations into several steps. First of all, you have to define and implement a theoretical model of the simulation. This model includes all necessary physical equations and approximations [14]. In a second step you have to gather data in order to trigger your simulation. This is a fundamental step of every numerical simulation [12]. Eventually, you conduct new observations. The new collected data is not only used to trigger a new simulation, but it is also used to verify your model: “Much of the improvement in NWP and in the statistical interpretation system can be tracked by the verification of the weather element guidance.” [15]

B. Physical and Mathematical Fundamentals

The fundamental necessary physical equations used in almost every weather model are the equations of motion and momentum, the equations for conservation of mass and the equation for conservation of energy, especially the first law of thermodynamics. The effects on physical processes by oceans, clouds and mountains (e.g. absorption of carbon dioxide or heat) are used to improve the weather models. [8,12,14]

However, the majority of these equations are partial differential equations. Bjerkenes describes the numeric problem as followed: “The task then consist of integrating a system with six partial differential equations with six unknowns [...]. There can be no question of a strictly analytical integration of the system of equations. [...] Graphical or mixed graphical and numerical methods are required to solve the task.” [12] Which numerical methods are used depends on the exact target of the model and the underlying experience of the people who implement the model. The WRF model uses Taylor series as one method for numerical approximation [14]. The COSMO model, an European project initialized by Deutscher Wetterdienst [16], uses Runge-Kutta methods and the so-called leapfrog method. Nevertheless, a variety of other numerical methods for solving differential equations are used in practice [14].

C. Parametrization

All physical and chemical processes which are not simulated with suitable equations for any reason (e.g. complexity, missing knowledge, smallness) are expressed by additional parameters for the simulation [14]. Typical examples are radiation, the effect of vegetation and soil temperature [17].

D. Data Assimilation

ECWMF uses a variety of different data sources to determine the current state of atmosphere for their predictions, amongst others satellites, aeroplanes, ships, buoys, registering balloons and radar stations [18]. Before this data can be used for simulations, it has to be assimilated. Present and past observations are combined with the results of former simulations and short range forecasts [19]. This seems to be simple but has a strong mathematical background [20].

V. ACCURACY

It is certainly uncontroversial that weather forecasts have become better for the last decades. This section examines how exact computer supported weather forecasts are and how scientists try to improve them for the future. One should consider that weather forecasts are not only important for normal people to decide whether they need an umbrella or not, but they have an significant importance for shipping traffic, air traffic, agriculture, police, fire brigade and technical emergency services [21, p. 1].

Accuracy of weather predictions is a vague term. It depends on the predicted phenomenons someone wants to consider. In Japan meteorologists have to predict the opening of cherry blossom buds for each region because Japanese celebrate this day with big events [21, p. 3]. More general weather variables are temperature, wind direction, wind speed, sky cover, ceiling height, precipitation, visibility, probability of thunder storms and snowfall amount [22].

A. Calculation

Depending on the weather variable different calculations are needed to measure the error of numerical weather predictions. The amount of rain has to be handled different than the probability of rain.

1) Numerical/Continuous Variables: The error of variables with numerical values such as the amount of rain or the air pressure can be calculated by using the mean absolute error (mae) or the root mean square error (rsme) [23, p. 2]:

$$\text{mae} = \frac{1}{n} \sum_{i=1}^{n} |e_i| \quad (1)$$

$$\text{rsme} = \sqrt{\frac{1}{n} \sum_{i=1}^{n} e_i^2} \quad (2)$$

$e_i$ stands for the absolute error in observation number $i$. rsme has the advantage that models with many outliers are ranked lower, so a model which always predicts the maximum temperature with an error of 0.5°C will be ranked better than
a model which nearly always predicts correctly, but sometimes is wrong with an error of 5°C.

2) Dichotomous Variables: Dichotomous means that a variable can have two states. One example is the forecast if it will rain. Therefore you have to determine a threshold (e.g., 1mm/day) which separates the two events rain and no rain. At the beginning of your verification you have to create a so-called contingency table:

### Table I
**CONTINGENCY TABLE**

<table>
<thead>
<tr>
<th>Observation</th>
<th>yes</th>
<th>no</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Forecast</td>
<td>hits</td>
<td>misses</td>
<td>correct negatives</td>
</tr>
<tr>
<td></td>
<td>yes</td>
<td>no</td>
<td>observed yes no</td>
</tr>
<tr>
<td>Total</td>
<td>yes</td>
<td>no</td>
<td>total</td>
</tr>
</tbody>
</table>

With this table you can calculate Bias score (BIAS) and Equitable threat score (ETS):

\[
BIAS = \frac{\text{hits} + \text{false alarms}}{\text{hits} + \text{misses}}
\]

\[
ETS = \frac{\text{hits} - \text{hits}_{\text{random}}}{\text{hits} + \text{misses} + \text{false alarms} - \text{hits}_{\text{random}}}
\]

\[
\text{hits}_{\text{random}} = \frac{(\text{hits} + \text{misses})(\text{hits} + \text{false alarms})}{\text{total}}
\]

Already its name implicates that the Bias score indicates if a model is biased or unbiased, i.e. if it predicts an event less often than observed (BIAS less than 0) or more often than observed (BIAS greater than 0). ETS ranges from \(-\frac{1}{2}\) to 1 with a perfect value of 1. It considers random hits and is an appropriate score to compare different models for different regimes. [24]

BIAS is often used along with ETS and can be used to improve the informative value of ETS [25].

3) Probabilistic Variables: There are several scores for calculating a score for probabilistic variables such as the probability of rain in percent [24]. One score, which is easy to understand, is the Brier score (BS) [26]:

\[
BS = \frac{1}{n} \sum_{i=1}^{n} (P_i - O_i)^2
\]

“The Brier score is the mean squared error in probability space.” [24] \(P_i\) is the predicted probability and \(O_i\) is either 0 (not observed) or 1 (observed). It obviously ranges from 0 to 1 with 0 as the best score. An example can be seen in Table IV which is explained in section V-C.

B. Wet Bias

In order to determine the accuracy of weather predictions one might analyse the accuracy of on-line services or TV forecasts. What is quite unknown is that TV weathermen make intentionally wrong predictions. They tend to predict a higher probability of rain. This effect is called wet bias. It is a psychological effect for the audience. If the weatherman predicts rain and it keeps dry, people link the missing rain to a positive feeling. However, if the weatherman predicts a low probability of rain, but it rains, people relied on this forecast and left their umbrellas at home, they tend to blame the weatherman. A similar effect can be observed on predictions of hurricanes or other natural disasters for the reason of safety. [27]

A study showed that forecasters from “The Weather Channel”, providing weather forecasts via television in the USA and via weather.com, tended to exaggerate the probability of precipitation if they actually would predict a probability below thirty or above ninety percent. A fifty percent probability was avoided. [28]

More about reasons why forecasts in general fail can be found in Nate Silver’s book The signal and the noise: Why so many predictions fail—but some don’t [29].

C. Accuracy of Meteorological Services

According to the British Met Office 85.2% of minimum temperature forecasts and 90.6% of maximum temperature forecasts are accurate with a deviation of 2°C on the next day. The prediction of rain on the current day succeeded with a rate of 73.3%. “[A] three-day forecast today is more accurate than a one-day forecast in 1980.” [30]

The German Weather Service (Deutscher Wetterdienst) declares that the average temperature for the next day can be predicted with a root mean square error between 1.0°C and 1.1°C [21, p. 6]. Figure 2 shows the accuracy of the average temperature forecast depending on the number of days ahead.

![Figure 2. Root Mean Square Error of Average Temperature Predictions](Adapted from [21, p. 6])

### Table II
**ACCURACY OF 24 HOURS PREDICTIONS IN 04/15 (DATA FROM [32])**

<table>
<thead>
<tr>
<th>Meteo. Agency</th>
<th>ECMWF</th>
<th>JMA</th>
<th>NCEP</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean sea-level pressure (hPa)</td>
<td>0.79</td>
<td>1.07</td>
<td>1.05</td>
</tr>
<tr>
<td>850 hPa temperature (K)</td>
<td>0.68</td>
<td>0.79</td>
<td>1.07</td>
</tr>
</tbody>
</table>

Table II summarizes the root mean square error of three leading meteorological agencies for a 24 hours prediction: The European Centre for Medium-Range Weather Forecasts (ECMWF), the Japan Meteorological Agency (JMA) and the
Table III

ACCURACY OF 264 HOURS PREDICTIONS IN 04/15 (DATA FROM [32])

<table>
<thead>
<tr>
<th>Meteo. Agency</th>
<th>ECMWF</th>
<th>JMA</th>
<th>NCEP</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean sea-level pressure (hPa)</td>
<td>7.19</td>
<td>7.76</td>
<td>7.59</td>
</tr>
<tr>
<td>850 hPa temperature (K)</td>
<td>3.82</td>
<td>3.64</td>
<td>4.05</td>
</tr>
</tbody>
</table>

National Centers for Environmental Prediction (NCEP) from the USA. In a height of 850 hPa ECMWF could predict the temperature with a root mean square error of only 0.68°C. The ISO standard atmosphere has a mean sea-level pressure of 1013.25 hPa [31]. Sea-level pressure could be predicted best by ECMWF with a root mean square error of 0.79 hPa. [32] Table III encapsulates the same data for a 360 hours prediction. Both data sets are valid for the northern hemisphere without the tropical zone (20°N - 90°N).

Table IV shows the Brier scores of several American weather forecast providers predicting the probability of precipitation for the next day. You can interpret the Brier score of a single forecaster as the probability of a wrong event prediction. The lower the Brier score the better. Section V-A3 describes how the Brier score is calculated.

Table IV

BRIER SCORES OF ONE-DAY PROBABILITY OF PRECIPITATION FORECASTS IN 2012/2013 (ADAPTED FROM [33])

<table>
<thead>
<tr>
<th>Rank</th>
<th>Provider</th>
<th>Brier Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Schneider Electric</td>
<td>0.1115</td>
</tr>
<tr>
<td>2</td>
<td>Weather Underground</td>
<td>0.1209</td>
</tr>
<tr>
<td>3</td>
<td>CustomWeather</td>
<td>0.1217</td>
</tr>
<tr>
<td>4</td>
<td>National Weather Service</td>
<td>0.1242</td>
</tr>
<tr>
<td>5</td>
<td>The Weather Channel</td>
<td>0.1315</td>
</tr>
</tbody>
</table>

D. Enhancement by Model Output Statistics

Model Output Statistics is a regression method “for a numerical weather prediction model, statistical relations between model-forecast variables and observed weather variables, used for either correction of model-forecast variables or prediction of variables not explicitly forecast by the model.” [34]

MOS compensates the low resolution of numerical weather simulations. Even high resolution regional NWP systems in current research have a horizontal resolution between two and five kilometres with the ability to simulate about 50 to 60 vertical layers up to about twenty kilometres [35]. In general, operational NWP systems have a horizontal grid resolution between ten and one hundred kilometres [36].

By using statistical data MOS corrects the output of NWP simulations and gives a sensible form to the output if the model itself frequently makes wrong predictions [37,38].

Furthermore MOS helps to interpret the output of NWP simulations. For example when the model predicts a certain relative humidity the statistical data can derive a value for rain probability [39].

E. Limitations

There are three main factors producing errors in NWP:

1) Erroneous or Insufficient Input Data: Every numerical simulation needs a set of input data to describe the initial state. The more data is provided the better. To enable the perfect numerical weather prediction one would have to measure the temperature, pressure etcetera of every single point of the air which is obviously impossible. Additionally, errors in the measurement, made by humans or caused by technical reasons, make every simulation imprecise.

According to [40] “[...] new (and accurate) observing systems, which measure the variables we need under all weather conditions, are the best way to improve NWP forecasts. Improvements in computers (which may allow higher horizontal and vertical resolution) and in the parameterizations of physical processes within the models will help, but to a lesser degree than new observing systems.”

2) Errors in the Model: A model is “[a] simplified description, especially a mathematical one, of a system or process, to assist calculations and predictions” [41], which means that every model has some deficits. As described in IV-B NWP needs numerical approximations, so every NWP is imprecise through its concept. Another reason for errors in the model is the lack of understanding in weather processes [12].

One should also consider that models are implemented as a kind of software. This is why bugs play also a role when models fail in their prediction.

3) Butterfly Effect: In 1963 Edward N. Lorenz described the fact that very small modifications of the initial conditions can lead to totally different solutions in numerical weather prediction [42]. To compensate this Butterfly effect ECMWF runs several parallel forecasts with slightly different initial parameters [19]. This technique is called multi-analysis ensemble. Another approach is the multi-model ensemble where different models share the same input data. [17]

VI. CONCLUSION

The wish to predict the weather is very old, as seen in section III, and is still present. A tremendous improvement of forecast accuracy has been showed in section V. Further improvements in meteorology, more statistical data, better observations and access to higher computational power will lead to even better forecasts in the future.

Future research will deal with hard to predict phenomenons like thunderstorms [43]. Nowadays, uncertainty is and probably will be an accepted variable in numerical weather prediction. Weather forecasts are a good example that nobody can predict the future. However, weather forecasts prove that meteorologists can at least make good assumptions about the future.